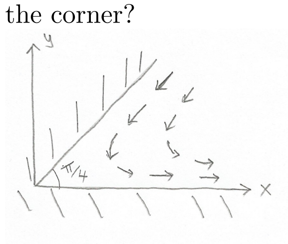
SNAP 2017. Laplace's equation and conformal maps.

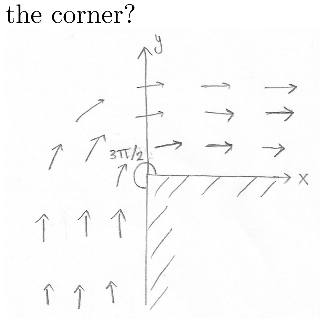
Problem Set 3

1. In lecture, we found a complex potential for a fluid flow around a corner of angle $\pi/2$.

   (a) Find a complex potential for the fluid flow when the corner has angle $\pi/4$ as shown. What is the speed of the fluid flow as you approach the corner?

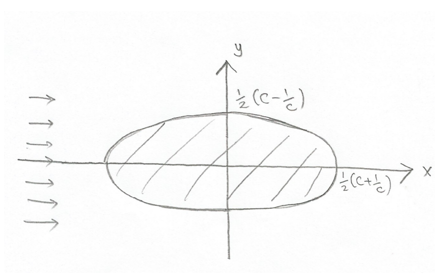
   (b) Find a complex potential for the fluid flow when the corner has angle $3\pi/2$ as shown. What is the speed of the fluid flow as you approach the corner?

2. Suppose a uniform horizontal flow of speed V encounters a cylindrical obstacle whose cross section is the ellipse

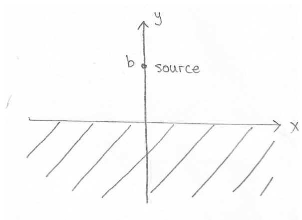
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a = \frac{1}{2}(c + 1/c),\ b = \frac{1}{2}(c - 1/c),$$

   for a constant $c > 1$. Find the complex potential describing the flow.

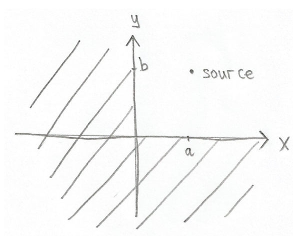
   Hint: first consider the cylinder with circular cross section of radius c

3. Suppose a fluid flow in the upper half plane is given by a source at a distance b as shown. Find a complex potential for the fluid flow.

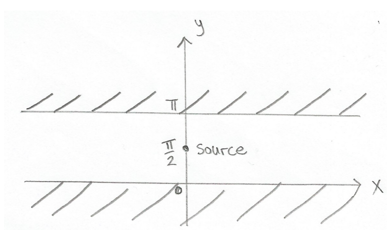


Hint: consider a fluid flow on the whole complex plane. Add a "mirror" source.

4. Suppose a fluid flow in the first quadrant is given by a source located at (a, b). Find a complex potential for the fluid flow.



5. Consider a fluid source at the point $(0, \pi/2)$ in a infinite channel of height π as shown. Determine the complex potential for the fluid flow.



Hint: use a conformal map

6. Fix a complex number β and a real number $r_0 > 0$. Suppose that the image of the disk $|z-\beta| \leq r_0$ under the Joukowsky map $w(z) = \frac{1}{2}(z+1/z)$ defines an airfoil shaped domain in the complex plane which contains the set $[-1, 1] \times \{0\}$. Show that the complex potential for the horizonal flow past the airfoil is a constant multiple of

$$f(w) = \frac{w - \beta + \sqrt{w^2 - 1}}{r_0} + \frac{r_0(w - \beta - \sqrt{w^2 - 1})}{\beta^2 + 1 - 2\beta w}.$$

7. Suppose a uniform horizontal flow of speed V encounters a hurdle of height b and negligible width. Find the complex potential describing the flow.